The Lancaster Space Gun – Atmospheric Drag

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This paper is one in a series that answer specific questions identified in the 'Introduction Assessment' paper.

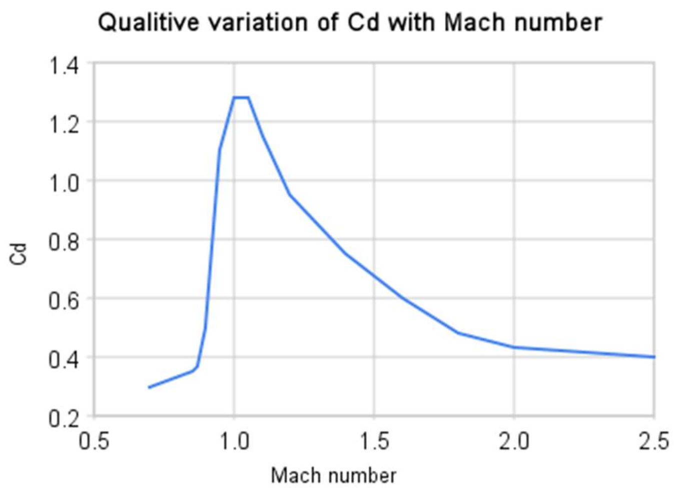
Q: How does atmospheric drag affect speed?

Drag depends heavily on the shape on the projectile. We can investigate some of the characteristics however to see how they impact the coasting phase. Two particular relevant aspects of drag are parasitic drag and wave drag, but these can be combined into NASA's drag equation[1]:

$$Drag\ (force) = coefficient\ \times air\ density\ \times \frac{velocity^2}{2} \times area$$

…although considerable complexity and implementation details such as nose shape are hidden in the coefficient, we can see that the drag is proportional to the square of the velocity. This means that any increase in muzzle velocity required to compensate for atmospheric drag must also allow for the *squared* increase in drag.

For simplicity we will assume an aerofoil shaped projectile which has a coefficient of about 0.05[2]. The coefficient isn't quite fixed as air behaves slightly differently supersonically:



..which suggests the coefficient may be as much as doubled at higher mach numbers, so we will use a working coefficient for an aerofoil of 0.1.

Taking the case of a launch at sea level (air density 1.2kg/m^3) and a cross section area of 0.78m^2 (assumption A3 – projectile diameter 0.5m) then:

[1] https://www.grc.nasa.gov/WWW/K-12/airplane/drageq.html

[2] https://www.grc.nasa.gov/WWW/K-12/airplane/shaped.html

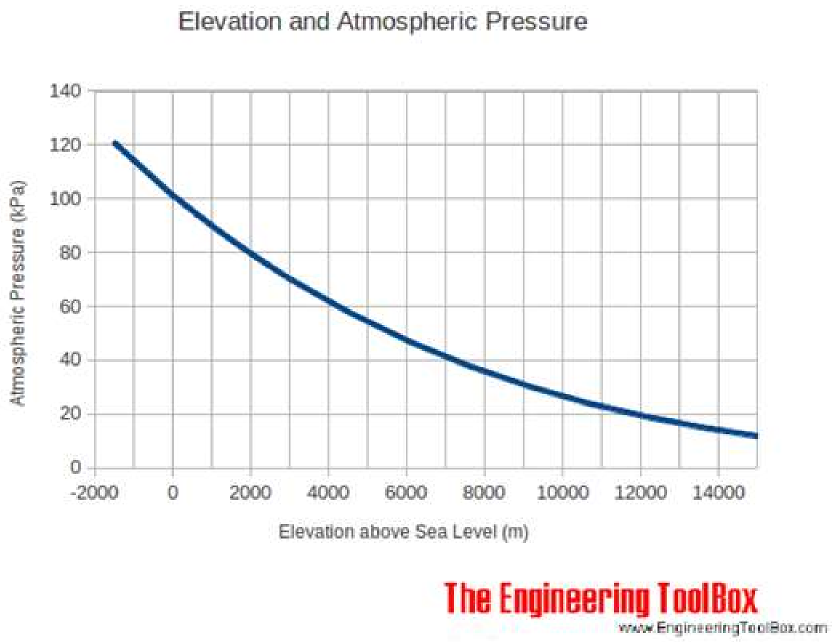
$$Drag\ (force) = 0.1\ \times 1.2\ \times \frac{(8.3\ \times\ 10^3)^2}{2} \times 0.78 = 3.2 \times\ 10^6 N$$

What difference does this make to velocity?

$$F = ma$$

$$deceleration = \frac{F}{m} = \frac{3.2\ \times\ 10^6}{500} = 6.4\ \ \times\ 10^3 ms^{-2}$$

Obviously this is significant as, with a vertical component of 2.8km/s, the projectile will spend several seconds in the atmosphere:



Later questions will have to consider how much time the projectile spends in the atmosphere. However using this working value, we can see that the muzzle velocity would have to be considerably increased to compensate for this speed loss, and that in turn increases the drag.

If we take an average of atmospheric density for the first second (around 1kg/m^3) and increase the muzzle velocity to compensate for the drag, we get:

$$Drag\ (force) = 0.1\ \times 1.0\ \times \frac{(14.7\ \times\ 10^3)^2}{2} \times 0.78 = 8.4 \times\ 10^6 N$$

...that is to say, more than twice as much. This in turn carries a higher deceleration, which must be compensated for, and so on.

This implies that there is ***no feasible muzzle velocity that can launch a projectile to orbit at this altitude with this drag coefficient and effective cross-section area***.

However if all three can be modified to reduce the drag to a tenth of the value here then this would appear more feasible.

For example using a 'Sears-Haack' body[3] shape with a length of 5m may be sufficient. Using the Sears-Haack body cross-section area equation[4] with *x* at 0.5 as the central section:

[3] https://en.wikipedia.org/wiki/Sears–Haack_body

$$Cross\ Section\ Area = \frac{16\,V}{3\,L\,\pi}\sqrt{(4\ \times \frac{1}{2} \times \left(1 - \frac{1}{2}\right))^3} = \frac{16\ \times 0.5}{3\ \times 5\ \times\ \pi} = 0.17m^2$$

The drag coefficient of the Sears-Haack body is given by[5]:

$$coefficient = \frac{24\,V}{L^3} = \frac{24\ \times 0.5}{5^3} = 0.096$$

This is similar to the aerofoil, and suggests that a longer, thinner body would be better, but using this feature set:

$$Drag\ (force) = 0.096\ \times 1.0\ \times \frac{(8.3\ \times\ 10^3)^2}{2} \times 0.17 = 5.6\ \times\ 10^5 N$$

which gives a deceleration of:

$$deceleration = \frac{F}{m} = \frac{5.6\ \times\ 10^5}{500} = 1.1\ \ \times\ 10^3 ms^{-2}$$

..and this appears to be more manageable. If we feed this back into the required muzzle velocity as, say, 10km/s, then:

$$Drag\ (force) = 0.096\ \times 1.0\ \times \frac{(10\ \times\ 10^3)^2}{2} \times 0.17 = 8.2\ \times\ 10^5 N$$

$$deceleration = \frac{8.2\ \times\ 10^5}{500} = 1.6\ \ \times\ 10^3 ms^{-2}$$

…which suggests that with a suitable projectile shape, the atmospheric drag can be compensated for by increasing muzzle velocity significantly but not overwhelmingly. Longer and more slender projectiles would reduce drag further, but there are likely engineering (flex, rear loads under acceleration, wind sheer) flight problems with very long projectiles.

[4] https://www.fxsolver.com/browse/formulas/Sears–Haack+body+%28cross+sectional+area%29

[5] https://www.fxsolver.com/browse/formulas/Sears–Haack+body+%28Drag+Coefficient+related+to+the+Volume%29